Secondary Math 3H SLO Review

Key

Name _____ Date _____ Period _____

Perform the indicated operation.

1. $(-8x^2+2x+1)+(10x^2-x)$

$2x^2+x+1$

2. $(5x^2+7x-8)-(7x^3+2x^2+3x)$

$5x^2+7x-8-7x^3-2x^2-3x$

$-7x^3+3x^2+4x-8$

3. $(-2y^2+3y-1)(y^2+2y+4)$

$-2y^4-4y^3-8y^2+3y^3+6y^2+12y$
$-y^2-2y-4$

$-2y^4-y^3-3y^2+10y-4$

4. $(x-2)(x^2+3x+4)$

$x^3+3x^2+4x-2x^2-6x-8$

x^3+x^2-2x-8

Use the given functions to find the following.

$f(x)=x^2+1$ *and* $g(x)=2x-3$

5. Find $h(x) = (f-g)(x)$.

$(x^2+1)-(2x-3)$

$x^2+1-2x+3 = x^2-2x+4$

6. Evaluate $f(-3) \cdot g(y-2)$

$f(-3) = (-3)^2+1 = 10$

$g(y-2) = 2(y-2)-3 = 2y-4-3 = 2y-7$

$f(-3)\cdot g(y-2) = 10\cdot(2y-7) = 20y-70$

For the given polynomials determine which of the binomials listed are factors.

7. $f(x) = x^3-x^2-5x-3$

a. $x+1 \rightarrow x=-1$ $f(-1) = (-1)^3-(-1)^2-5(-1)-3 = -1-1+5-3 = 0$ yes

b. $x-1 \rightarrow x=1$ $f(1) = (1)^3-(1)^2-5(1)-3 = 1-1-5-3 = -8$ no

c. $x-3 \rightarrow x=3$ $f(3) = (3)^3-(3)^2-5(3)-3 = 27-9-15-3 = 0$ yes

Without graphing, determine the end behavior of each, then write the end behavior as a limit.

$\lim_{x\to-\infty} f(x) =$ $\lim_{x\to\infty} f(x) =$

8. $f(x) = 2x^2-8x+6$ (positive, even)

$\lim_{x\to\infty} f(x) = \infty$

$\lim_{x\to-\infty} f(x) = \infty$

9. $f(x) = -x^3-x^2-3$ (negative, odd)

$\lim_{x\to\infty} f(x) = -\infty$

$\lim_{x\to-\infty} f(x) = \infty$

Without graphing, determine the **number** of zeros for each of the following polynomials.

10. $f(x) = x^4-2x^2-5x+6$

four zeros

11. $f(x) = -x^3-x^2-5x-3$

3 - zeros

Identify the zeros, their multiplicity, determine whether they touch or cross the x-axis at each zero

12. $f(x) = (x+1)(x-1)(x-3)$

Zeros	mult.	touch/cross
$(-1,0)$	1	cross
$(1,0)$	1	cross
$(3,0)$	1	cross

13. $f(x) = 6x^3(x^2-9)(x+2)$ → $(x+3)(x-3)$

Zeros	mult.	touch/cross
$(0,0)$	3	cross
$(3,0)$	1	cross
$(-3,0)$	1	cross
$(-2,0)$	1	cross

Factor each expression using the polynomial identities. Show work!

14. $27y^3+8$ (Sum of cubes)

$(3y+2)(9y^2-6y+4)$

15. x^2+6x+8

$(x+4)(x+2)$

Factor each expression over the complex numbers. Use identities if needed. Show work!

16. $4x^2+49$ Sum of Squares

$(2x+7i)(2x-7i)$

17. $x^2-10x+26$ use quad. formula to get zeros. Then write in factored form.

$a=1, b=-10, c=26$

$x = \frac{10 \pm \sqrt{(-10)^2-4(1)(26)}}{2(1)} = \frac{10 \pm \sqrt{-4}}{2} = \frac{10 \pm 2i}{2} = 5 \pm i$

$(x-5-i)(x-5+i)$

18. An auditorium has 30 rows with 10 seats in the first row, 12 seats in the second row, 14 seats in the third row, and so forth. How many seats are in the auditorium?

$n=30,\ a_1=10\ \ d=2 \qquad S=\frac{n}{2}(a_1+a_n) \qquad a_n=a_1+(n-1)d$

$a_{30} = 10+(30-1)2$

$a_{30} = 10+58$

$=68$

$S = \frac{30}{2}(10+68) =$

$= 15(78) =$ 1170 seats

Use long division to rewrite the expression.

19. $\frac{2x^4-x^3-4x-2}{x^2+x+1}$

$2x^2-3x+1+\frac{-2x-3}{x^2+x+1}$

$x^2+x+1 \overline{)\,2x^4-x^3+0x^2-4x-2}$

$-(2x^4+2x^3+2x^2)$

$-3x^3-2x^2-4x$

$-(-3x^3-3x^2-3x)$

x^2-x-2

$-(x^2+x+1)$

$-2x-3$

20. $\frac{x^3+2x^2-3x+4}{x+1}$

$x^2+x-4+\frac{8}{x+1}$

$x+1 \overline{)\,x^3+2x^2-3x+4}$

$-(x^3+x^2)$

x^2-3x

$-(x^2+x)$

$-4x+4$

$-(-4x-4)$

8

Perform the indicated operation. Simplify if possible

21. $\frac{x+5}{x-6} \cdot \frac{2x-12}{x^2-25}$ Factor & reduced $\frac{2(x-6)}{(x+5)(x-5)} = \frac{2}{x-5}$

22. $\frac{x^2+5x-14}{3x^3-6x^2} \cdot \frac{2x^2+6x}{x^2+10x+21}$ Factor & reduced $\frac{(x+7)(x-2)}{3x^2(x-2)} \cdot \frac{2x(x+3)}{(x+5)(x+7)} = \frac{2}{3x}$

Get common denominator

23. $\frac{2x}{x+5} - \frac{x}{x+8}$

$\frac{2x(x+8)}{(x+5)(x+8)} - \frac{x(x+5)}{(x+8)(x+5)} = \frac{2x^2+16x-x^2-5x}{(x+8)(x+5)}$

$= \boxed{\frac{x^2+11x}{(x+8)(x+5)}}$

24. $\frac{-6}{x-3} + \frac{5}{x-2}$

$\frac{-6(x-2)}{(x-3)(x-2)} + \frac{5(x-3)}{(x-2)(x-3)} = \frac{-6x+12+5x-15}{(x-3)(x-2)}$

$= \boxed{\frac{-x-3}{(x-3)(x-2)}}$

Perform the indicated operation. All answers should be reduced. Show work!

25. $\frac{x+3}{x-1} \cdot \frac{1-x}{x^2-9} = \frac{-(x-1)}{(x+3)(x-3)} = \boxed{\frac{-1}{x-3}}$

26. $\frac{9}{m+2} + \frac{8}{m-7}$

$\frac{9(m-7)}{(m+2)(m-7)} + \frac{8(m+2)}{(m-7)(m+2)} = \frac{9m-63+8m+16}{(m+2)(m-7)}$

$= \boxed{\frac{17m-47}{(m+2)(m-7)}}$

Solve the equation.

27. $\frac{3}{x+2} + \frac{6}{x^2+2x} = \frac{3-x}{x}$ $\quad x(x+2)$, $x \neq 0, x \neq -2$

$3x+6 = (3-x)(x+2)$ no solution

$3x+6 = 3x+6-x^2-2x$

$3x+6 = -x^2+x+6 \Rightarrow x^2+2x=0$

$x(x+2)=0$

$x=0 \quad x=-2$

28. $\frac{4x}{x+4} + \frac{5}{x-1} = \frac{15}{x^2+3x-4}$ $\quad (x+4)(x-1)$, $x \neq -4, x \neq 1$

$4x(x-1) + 5(x+4) = 15$

$4x^2-4x+5x+20 = 15$

$4x^2+x+5=0$

$x = \frac{-1 \pm \sqrt{(1)^2-4(4)(5)}}{2(4)} = \frac{-1 \pm \sqrt{1-80}}{8}$

$\boxed{x = \frac{-1 \pm \sqrt{79}\,i}{8}}$

29. $\sqrt{4x-23} - 3 = 2$

$+3 \quad +3$

$(\sqrt{4x-23})^2 = (5)^2$

$4x-23 = 25$

$+23 \quad +23$

$4x = 48$

$\boxed{x=12}$

30. $\sqrt{2x+3} - 7 = 0$

$+7 \quad +7$

$(\sqrt{2x+3})^2 = (7)^2$

$2x+3 = 49$

$2x = 46$

$\boxed{x=23}$

Solve for the specified variable.

31. $\frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}}$, *solve for* M_2

$\left(\frac{r_1}{r_2}\right)^2 = \left(\sqrt{\frac{m_2}{m_1}}\right)^2$

$m_1 \cdot \frac{r_1^2}{r_2^2} = \frac{m_2}{m_1} \cdot m_1 \Rightarrow \boxed{\frac{m_1 r_1^2}{r_2^2} = M_2}$

32. $\sqrt{b^2-4ac} = k$, *solve for* c

$(\sqrt{b^2-4ac})^2 = (k)^2$

$b^2-4ac = k^2$

$-b^2 \quad -b^2$

$\frac{-4ac}{-4a} = \frac{k^2-b^2}{-4a} \Rightarrow \boxed{c = \frac{k^2-b^2}{-4a}}$

odd: $f(-x) = -f(x)$
even: $f(x) = f(-x)$

33. Determine if the functions are even, odd or neither.

a. $f(x) = -3x^4$

even $f(x) = f(-x)$

$f(-x) = -3(-x)^4$

$= -3x^4$ even

b. $f(x) = 2x^3 + 5x$

$f(-x) = 2(-x)^3 + 5(-x)$

$= -2x^3 - 5x$

$-f(x) = -(2x^3 + 5x)$

$= -2x^3 - 5x$

odd

c. $f(x) = 6x^3 - x^2$

$f(-x) = 6(-x)^3 - (-x)^2$

$= -6x^3 - x^2$

$-f(x) = -(6x^3 - x^2)$

$= -6x^3 + x^2$

neither

34. Graph the given function and determine the following:

$f(x) = x^3 - 2x^2 - 3x$

Relative maximum: $(-.535, .879)$ → max value

Relative minimum: $(1.869, -6.065)$ → min value

x-intercepts: $(-1, 0)(0, 0)(3, 0)$

y-intercept: $(0, 0)$

On what interval(s) is the function increasing? $(-\infty, -.535)(1.869, \infty)$

On what interval(s) is the function decreasing? $(-.535, 1.869)$

35. Graph the given function and determine the following:

$f(x) = \dfrac{x^2 - 9}{x^2 - 2x - 15}$ $\dfrac{(x+3)(x-3)}{(x-5)(x+3)}$

Domain: $x \neq 5, x \neq -3$

Range: $y \neq 1$

hole at $x = -3$

Asymptotes: $x = 5$, $y = 1$

Points of discontinuity: $\left(-3, \frac{3}{4}\right)$

36. Find the average rate of change on the specified interval.

The average high temperature per month in the city of South Jordan for the year 2012 is shown in the table below. Determine the average rate of change from September to December.

Month	Temp °F	Month	Temp °F
Jan	41	July	96
Feb	47	Aug	93
March	57	Sept 9	83
April	65	Oct 10	68
May	76	Nov 11	53
June	88	Dec 12	42

$\dfrac{42 - 83}{12 - 9} = \dfrac{-41}{3}$

$-13.67°$ per month

For the sequence, write the rational equation that models the relationship between the term in the sequence and its value.

37. $\frac{-1}{2}, 0, \frac{1}{4}, \frac{2}{5}, \frac{1}{2}, \frac{4}{7}, \ldots$

$-\frac{1}{2}, \frac{0}{3}, \frac{1}{4}, \frac{2}{5}, \frac{3}{6}, \frac{4}{7}$

$\frac{n-2}{n+1}$

n	f(n)
1	$\frac{-1}{2}$
2	0
3	$\frac{1}{4}$
4	$\frac{2}{5}$

$f(n) = \frac{n-2}{n+1}$

Rewrite equation in exponential form.

38. $\log_3(3x-2) = 3$

$3^3 = 3x - 2$

Find the inverse of each function.

39. $f(x) = -2\sqrt{x-5}$

$\frac{x}{-2} = \frac{-2\sqrt{y-5}}{-2}$

$\left(\frac{x}{-2}\right)^2 = \left(\sqrt{y-5}\right)^2$

$\frac{x^2}{4} = y - 5$

$+5 \quad +5$

$\frac{x^2}{4} + 5 = y$

$f^{-1}(x) = \frac{x^2}{4} + 5$

40. $f(x) = \frac{\sqrt{x+3}}{6}$

$\Rightarrow 6x = \frac{\sqrt{y+3}}{6} \cdot 6$

$(6x)^2 = \left(\sqrt{y+3}\right)^2$

$36x^2 = y + 3$

$-3 \quad -3$

$36x^2 - 3 = y$

$f^{-1}(x) = 36x^2 - 3$

$x \geq -3$

Condense the expressions to a single logarithm.

41. $2\log x + \log y - 5\log z$

$\log\left(\frac{x^2 y}{z^5}\right)$

42. $4\log_3 x - 2\log_3 y + \log_3 z$

$\log_3\left(\frac{x^4}{y^2 z}\right)$

Find the value of each expression. Round to the nearest hundredth.

43. $\log_8 265 = \frac{\log 265}{\log 8} \approx 2.68$

Rewrite the equation in logarithmic form.

44. $6^x = 21$

$\log_6 21 = x$

Find the exact value of each expression using the unit circle. (No decimals!)

45. $\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$ 46. $\sin\frac{5\pi}{3} = -\frac{\sqrt{3}}{2}$

47. Find all solutions in the interval $[0, 2\pi)$ for

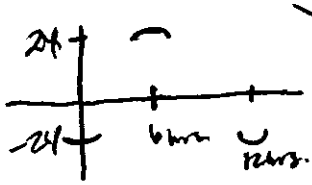
$-8\sin x + 4 = 0$. **Show all work!**

$-4 \quad -4$

$\frac{-8\sin x}{-8} = \frac{-4}{-8}$ $\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

48. Low tide is at 9:12 am and high tide is at 3:12 pm. The water level varies 48 inches between low and high tide. Write a cosine function to represent the change in water level. Show work!



$y = a\cos b(x-c)$ $a = 24$ in period is 12 hrs.

$P = \frac{2\pi}{b}$

$12 = \frac{2\pi}{b}$ $b = \frac{\pi}{6}$

$y = -24\cos\left(\frac{\pi}{6}x\right)$

Find the area of the triangle. Show work!

49. A = 52°, b = 14 m, c = 21 m $A = \frac{1}{2}bc\sin A$

$A = \frac{1}{2}(14)(21)\sin 52° = 115.8\ m^2$

Use the Law of Sines to solve the triangle. Show work!

50. A = 49°, a = 32, b = 28 $\frac{\sin 49°}{32} = \frac{\sin B}{28}$

$B = \sin^{-1}\left(\frac{28\sin 49°}{32}\right) \approx 41.3°$

$\angle B = 41.3°$

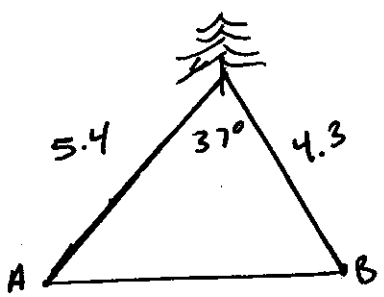
$\angle C = 180° - 49° - 41.3° = 89.7°$

$C = 42.4$

$\frac{\sin 49°}{32} = \frac{\sin 89.7°}{c}$

$c = \frac{32\sin 89.7°}{\sin 49°} = 42.4$

51. Fire-lookout station A is 5.4 miles from the tallest tree in the forest. Fire-lookout station B is 4.3 miles from the tallest tree in the forest. The line of sight from the tree to Station A , and from the tree to station B forms and angle of 37 degrees, how far apart are the lookout stations from each other?



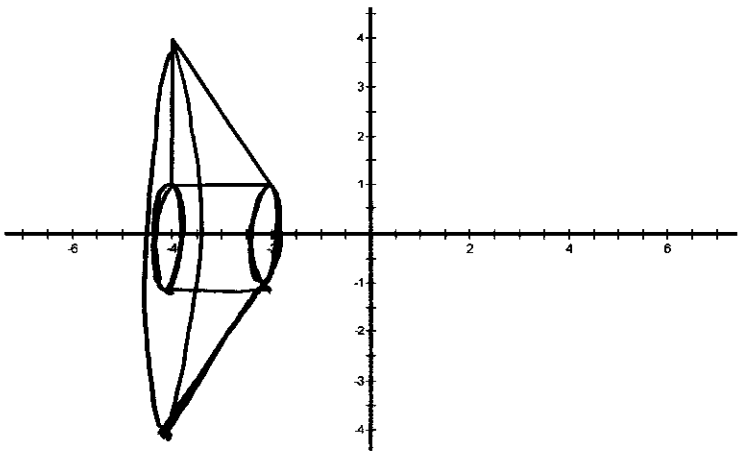
Law of Cosines

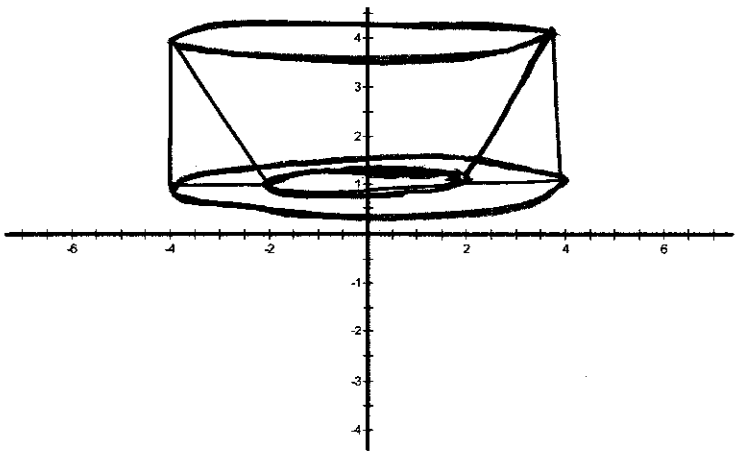
$c^2 = 5.4^2 + 4.3^2 - 2(5.4)(4.3)\cos 37°$

$c^2 = 10.56$

$c = 3.25$ miles apart

52. Draw the result of rotating the given shape around the a) x-axis b) the y-axis.



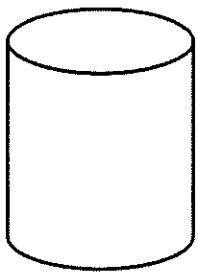


53. Given a cylinder, what two-dimensional shape will result from a

A) Horizontal slice? Circle

B) Vertical slice through both bases? Rectangle

C) Diagonal slice not through either base?
Ellipse



54. Your grandmother also needs some help in her yard. She would like to put some compost in her flower beds. One flower bed is shaped like a circle with a diameter of 7ft., the other is a rectangle with dimensions 3ft. by 6ft. The compost should be 3 inches deep. The compost is $4.25 per bag and covers 2 cubic ft. How many bags of compost will be needed to fill both flower beds?

$A = \pi r^2 \Rightarrow \pi(3.5)^2 = 38.48\ ft^2$

$A = 3 \times 6 = 18\ ft^2$ $\quad 18 + 38.48 = 56.48$

3 inches $= \frac{1}{4}$ ft

$(56.48)(\frac{1}{4}) = 14.12$

$14.12 \div 2 = 7.06 \Rightarrow$ 8 bags

55. Use the given table below to answer the following.

A) According to the table, which city has the highest population density?

B) According to the table, which city has the lowest population density?

City	Land Size (sq. miles)	Population(Based on 2010 census)
South Jordan	22	50,418
Riverton	12.6	38,753
Herriman	20	21,785
West Jordan	31	103,712

Density:

S.J. $\frac{50,418}{22} = 2291.4$

R.N. $\frac{38,753}{12.6} = 3075.6$

Herr. $\frac{21,785}{20} = 1089.25 \Rightarrow$ Lowest

W.J. $\frac{103,712}{31} = 3345.5$ — Highest

56. The principal wants to know how many students eat school lunch each day. The principal decided to have teachers in their 1st period classes ask the students if they eat school lunch each day. This is an example of what kind of sample? Cluster Sample

57. What is the parameter of interest in question 50? students who eat school lunch each day.

58. A yoga instructor wanted to test the results of a new yoga workout. She had one class continue to do the regular workout and another class tried the new workout. What is this an example of?

A) an experiment without a control group
B) an experiment with a control group
C) an observational study
D) a voluntary sample survey

59. If the yoga instructor decided to randomly select students from her classes and monitor their workouts and record flexibility each week for 5 weeks this would be an example of

A) an experiment without a control group
B) an experiment with a control group
C) an observational study
D) a voluntary sample survey

60. Americans consume a mean of 23.4 pounds of candy per year with a standard deviation of 5.2 pounds. If the distribution is normal, what is the interval that contains

A) 68% of the population? (18.2, 28.6)

B) 95% of the population? (13, 33.8)

C) 99.7% of the population? (7.8, 39)

61. A poll states that 56%, with a margin of error ±3%, of the population supports a new park. According to these statistics, what is the smallest percent of people who support the park?

53%

SLO Review cont. (Honors topics) Key

Name _____ Date _____ Period _____

Solve the following equations.

1. $2(x-1)^{\frac{4}{3}}+4=36$

$-4 \quad -4$

$\frac{2(x-1)^{4/3}}{2} = \frac{32}{2}$

$\left((x-1)^{4/3}\right)^{3/4} = (16)^{3/4}$

$x-1 = \pm 8$

$x = \pm 8 + 1$

$\boxed{x = 9, -7}$

2. $e^{-2x} + 2e^{-x} = 3$ (hint: use u-substitution)

let $u = e^{-x}$

$u^2 + 2u - 3 = 0$

$(u+3)(u-1) = 0$

$u = -3 \quad u = 1$

$e^{-x} \neq -3$

$e^{-x} = 1$

$\boxed{x = 0}$

3. Find the inverse of the function.

$f(x) = \log(x+4) - 7$

$x = \log(y+4) - 7$

$x+7 = \log(y+4)$

$10^{(x+7)} = y+4$

$\boxed{f^{-1}(x) = 10^{(x+7)} - 4}$

4. Use the unit circle to find the exact value of $\sec\frac{5\pi}{4}$.

$\sec\frac{5\pi}{4} = \frac{1}{\cos\frac{5\pi}{4}}$

$= \boxed{-\sqrt{2}}$

5. Determine the domain for the rational function.

$f(x) = \frac{x+2}{3x^2-20x-32}$ ← Factor!

$(3x+4)(x-8)$

$3x+4 \neq 0 \quad x-8 \neq 0$

$x \neq -4/3 \quad x \neq 8$

$\boxed{\text{Domain: } (-\infty, -4/3) \cup (-4/3, 8) \cup (8, \infty)}$

6. Use symmetry (or even odd properties) to find the exact values of sin Θ and cos Θ for

$\Theta = -\frac{4\pi}{3}$.

$\sin\left(-\frac{4\pi}{3}\right) = -\sin\left(\frac{4\pi}{3}\right) = \boxed{\frac{\sqrt{3}}{2}}$

$\cos\left(-\frac{4\pi}{3}\right) = \cos\left(\frac{4\pi}{3}\right) = \boxed{-\frac{1}{2}}$

7. What are the rectangular coordinates of the polar coordinates $(3, \frac{2\pi}{3})$?

$x = r\cos\theta \quad , \quad y = r\sin\theta$

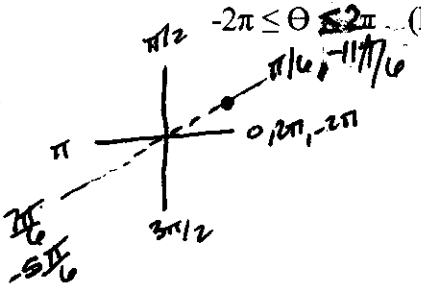
$x = 3\cos\frac{2\pi}{3} \qquad y = 3\cdot\sin\frac{2\pi}{3}$

$x = 3\cdot\frac{-1}{2} \qquad y = 3\cdot\frac{\sqrt{3}}{2}$

$x = -\frac{3}{2} \qquad y = \frac{3\sqrt{3}}{2}$

$\boxed{\left(-\frac{3}{2}, \frac{3\sqrt{3}}{2}\right)}$

8. Find all polar coordinates for the point $(2, \frac{\pi}{6})$ for $-2\pi \leq \Theta \leq 2\pi$. (hint there are 3 points)



$\boxed{(2, -\frac{11\pi}{6}), (-2, \frac{7\pi}{6}), (-2, -\frac{5\pi}{6})}$

9. Find the polar equation for the given Cartesian (rectangular) equation $y^2 = 3x$.

$y = r\sin\theta$, $x = r\cos\theta$

$(r\sin\theta)^2 = 3r\cos\theta$

$\frac{r^2\sin^2\theta}{r\sin^2\theta} = \frac{3r\cos\theta}{r\sin^2\theta} \Rightarrow$ $\boxed{r = \frac{3\cos\theta}{\sin^2\theta}}$

10. What is the polar (trig.) form of the complex number $z = 1 - \sqrt{3}\,i$? → 4th quadrant.

$r = \sqrt{(1)^2 + (-\sqrt{3})^2} = \sqrt{1+3} = 2$

$\theta = \tan^{-1}\left(-\frac{\sqrt{3}}{1}\right) = 300^\circ$ or $\frac{5\pi}{3}$

$\boxed{z = 2\left(\cos\left(\frac{5\pi}{3}\right) + i\sin\left(\frac{5\pi}{3}\right)\right)}$

11. Find the product of the complex numbers $z_1 = 2(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$ *and* $z_2 = 3(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$.

$z_1 \cdot z_2 = 2 \cdot 3\left(\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right)\right)$

$= 6\left(\cos\left(\frac{7\pi}{12}\right) + i\sin\left(\frac{7\pi}{12}\right)\right)$

12. Each time a ball bounces the height of the ball decreases after each bounce. If a ball is dropped 12 inches from the ground and after the first bounce reaches a height of 11.16 inches, and after each bounce the height decreases by the same percentage, what is the total distance the ball bounces when it comes to rest? (Hint: take into account the ball is going up then down)

sum of an infinite geometric series: $S = \frac{a}{1-r}$

$a = 12$, $r = \frac{11.16}{12} = .93$

$S = \frac{12}{1-.93} = 171.42$

$(171.42 \cdot 2) - 12 =$ $\boxed{330.86 \text{ inches}}$